Poster #WTh275 – Event-related EEG/MEG time-frequency analysis: Relating power and coherence across trials

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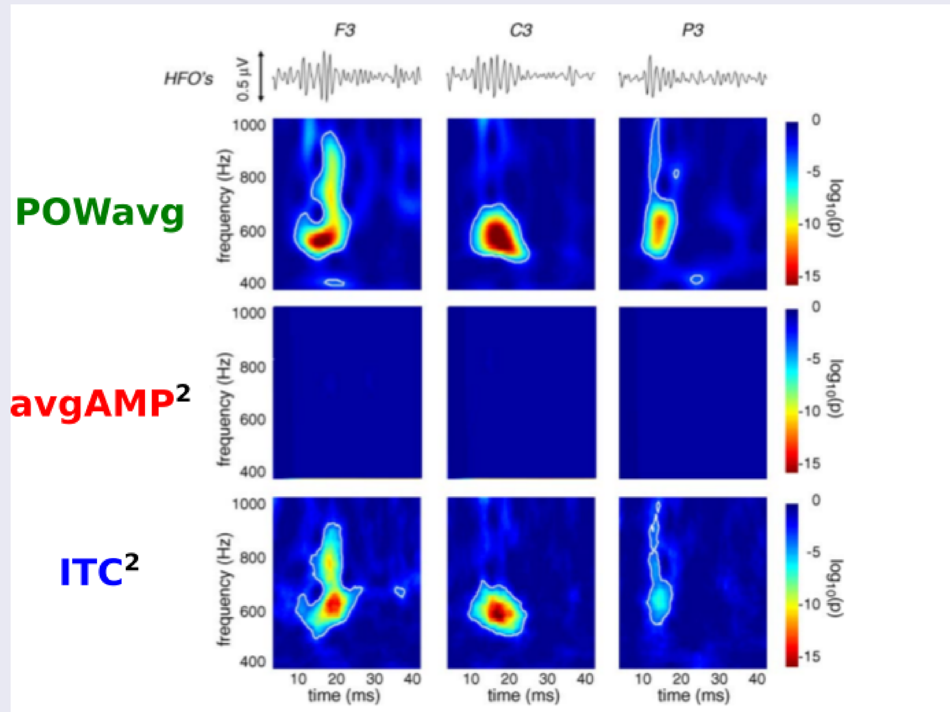
Introduction

Context:

- Interpretation of event-related responses in brain recordings: stimulus-evoked neuronal activity or stimulus-induced phase resetting of ongoing neuronal dynamics? [1, 2, 3, 4]
- Three measures from time-frequency transform (TFT): amplitude averaged across trials (**avgAMP**); inter-trial phase coherence (**ITC**); TFT power of evoked potential (**POWavg**).
- Sensitivity to different aspects: evoked responses for **avgAMP**, and induced responses for **ITC** and **POWavg**.
- Common sensitivity of **ITC** and **POWavg** to induced responses and overall similarity between both measures.



From [6]

Objective:

- Further investigatation of the relationship between **ITC** and **POWavg**.
- Theoretical calculations and simulation study.

Definitions

- TFT using S-transform: Band-pass filter/windowed Fourier transform with a window whose width decreases with increasing frequency [5], $T_x(t,f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(u)\,|f|\,e^{-\frac{f^2(u-t)^2}{2}}\,e^{-2i\pi f u}\,\mathrm{d}u$.
- **avgAMP** $= \frac{1}{N}\sum_{n=1}^{N} |T_{x_n}(t,f)|$.
- **ITC** $= |\frac{1}{N}\sum_{n=1}^{N} e^{i\theta_{x_n}(t,f)}|$.
- **POWavg** $= |T_{\overline{x_n}}(t,f)|^2$ with $\overline{x_n}(t) = \frac{1}{N}\sum_{n=1}^{N} x_n(t)$.

References

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[6] M. Valencia, M. Alegre, J. Iriarte, and J. Artieda. High frequency oscillations in the somatosensory evoked potentials (SSEP's) are mainly due to phase-resetting phenomena. *J. Neurosci. Methods*, 154:142–148, 2006.

Oscillatory signal

General model:

- N repetitions (trials).
- For each trial, cosine with amplitude Ω_n, frequency ν_n, and phase ϕ_n, i.e., $x_n(t) = \Omega_n \cos(2\pi\nu_n t + \phi_n)$.
- S-transform: $T_x(t,f) \approx \frac{\Omega}{2} e^{-\frac{1}{2}(2\pi)^2\left(1-\frac{\nu}{f}\right)^2} e^{i[\phi - 2\pi(f-\nu)t]}$.

First model:

- Constant frequency ($\nu_n = \nu_0$) and amplitude ($\Omega_n = \Omega_0$), varying phases $\phi_n \sim \mathrm{VonMises}(\phi_0, \kappa)$.
- **POWavg** = **avgAMP**$^2 \times$ **ITC**^{2}.

Second model

- Constant frequency ($\nu = \nu_0$), varying amplitude $\Omega_n \sim \mathcal{N}(\Omega_0, \tau_\Omega^2)$ and phase $\phi_n \sim \mathrm{VonMises}(\phi_0, \kappa)$.
- E(**POWavg**) = E(**avgAMP**$^2 \times$ **ITC**^{2}) + $O(1/N)$.

Third model

- Varying frequency $\nu_n \sim \mathcal{N}(\nu_0, \tau_\nu^2)$, amplitude $\Omega_n \sim \mathcal{N}(\Omega_0, \tau_\Omega^2)$ and phase $\phi_n \sim \mathrm{VonMises}(\phi_0, \kappa)$.
- E(**POWavg** − **avgAMP**$^2 \times$ **ITC**^{2}) $\neq O(1/N)$.

General relationship

- Brute force calculation shows that we have

$$\begin{aligned}
&\mathrm{E}(\mathbf{POWavg} - \mathbf{avgAMP}^2 \times \mathbf{ITC}^2) \\
&= \left|\mathrm{E}\left[e^{i\theta_x(t,f)}\right]\mathrm{E}\left[|T_x(t,f)|\right] + \mathrm{Cov}\left[e^{i\theta_x(t,f)}, |T_x(t,f)|\right]\right|^2 \\
&\quad - \left|\mathrm{E}\left[e^{i\theta_x(t,f)}\right]\right|^2 \mathrm{E}\left[|T_x(t,f)|\right]^2 + O\left(\frac{1}{N}\right).
\end{aligned} \quad (1)$$

- If $\mathrm{Cov}\left[e^{i\theta_x(t,f)}, |T_x(t,f)|\right] = 0$, then the right-hand side of the equation is equal to $O(1/N)$, i.e.,

$$\mathrm{E}(\mathbf{POWavg} - \mathbf{avgAMP}^2 \times \mathbf{ITC}^2) = O\left(\frac{1}{N}\right). \quad (2)$$

Simulation study

Data generation:

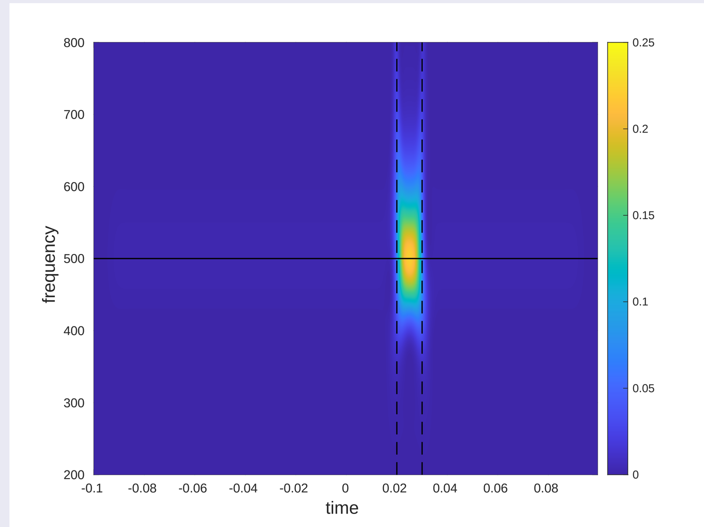
- $N = 300$ trials over $[-100, 100]$ ms time window, sampling rate $f_s = 2$ kHz.
- Model of oscillatory signal as above (cosine) with $\phi_n = \phi_n^{(i)}$ between 20 and 30 ms, $\phi_n = \phi_n^{(o)}$ otherwise (HFOs with phase resetting).

Parameter	Distribution	Parameters
Ω	$\mathcal{N}(\Omega_0, \tau_\Omega^2)$	$\Omega_0 = 1 \quad \tau_\Omega = 0.1$
ν	ν_0	$\nu_0 = 500$
$\phi_n^{(o)}$	$\mathrm{vonMises}\left[\phi_0, \kappa^{(o)}\right]$	$\phi_0 = 0 \quad \kappa^{(o)} = 0$
$\phi_n^{(i)}$	$\mathrm{vonMises}\left[\phi_0, \kappa^{(i)}\right]$	$\phi_0 = 0 \quad \kappa^{(i)} = 10$

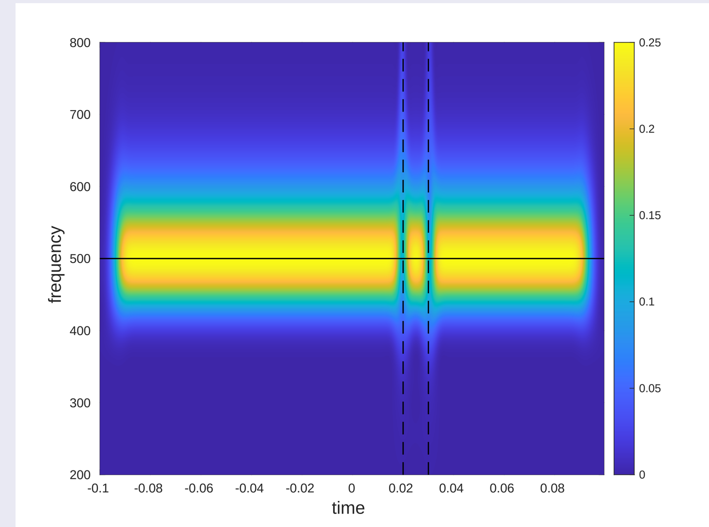
Simulation study (cont'd)

Results:

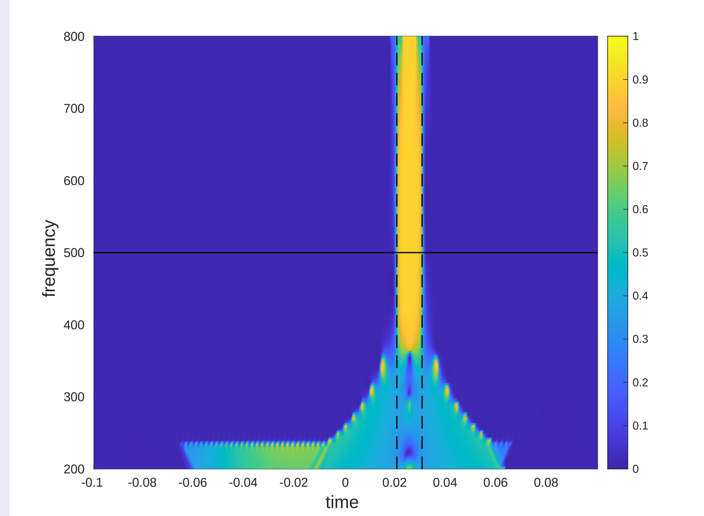
POWavg



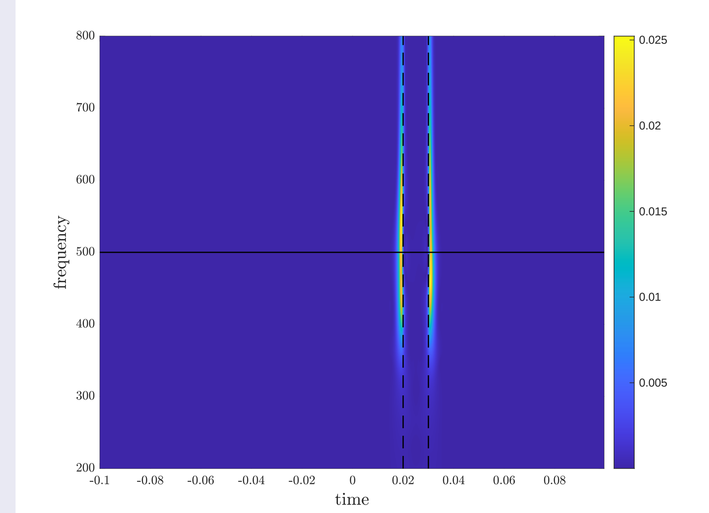
avgAMP^2



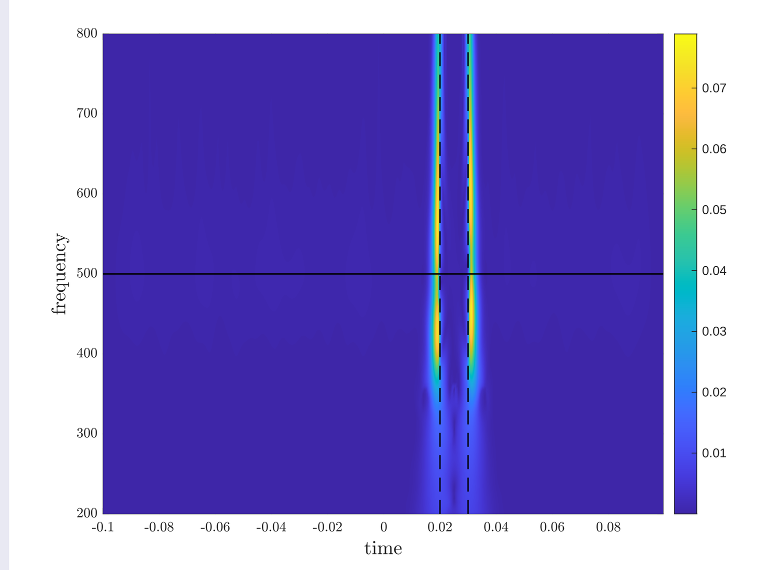
ITC^2



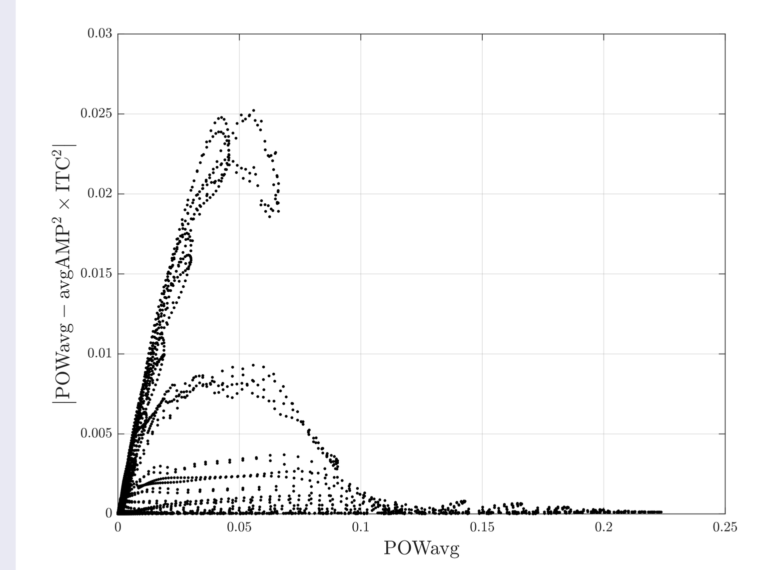
|**POWavg** − **avgAMP**$^2 \times$ **ITC**^{2}|



$\left|\mathrm{Cov}\left[e^{i\theta_x(t,f)}, |T_x(t,f)|\right]\right|$



Error as a function of **POWavg**



Discussion

- **avgAMP**, **ITC** and **POWavg** are connected through **POWavg** ≈ **avgAMP**$^2 \times$ **ITC**^{2}.
- Validation for pure oscillatory signal.
- Confirmation on simulations of HFOs with phase resetting.
- Similar results with continuous wavelet transforms (we tried Morse wavelets and analytic Morlet wavelets).
- Interested for more details? Keep an eye out for a potential paper (currently under review in *IEEE Transactions on Biomedical Engineering*).