Event-related EEG/MEG time-frequency analysis: Effect of noise on power (Poster #480)

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1. Introduction

- Time-frequency analysis of brain recordings during event-related protocols
- N trials, $x_n(t)$ recording from nth trial, corresponding time-frequency transform $T_{x_n}(t,f)$
- Two measures of power: the power of the transform for each single trial averaged across trials, avgPOW (Hari and Puce, 2017)

$$\mathrm{avgPOW}_{x_{1:N}}(t,f) = \frac{1}{N}\sum_{n=1}^{N} |T_{x_n}(t,f)|^2, \quad (1)$$

and the power of the time-frequency transform applied to the average response over all trials (Tallon-Baudry et al., 1996)

$$\mathrm{POWavg}_{x_{1:N}}(t,f) = |T_{\overline{x_n}}(t,f)|^2 = \left|\frac{1}{N}\sum_{n=1}^{N} T_{x_n}(t,f)\right|^2 \quad (2)$$

2. Theoretical developments

Time-frequency analysis and the S-transform

- Time-frequency analysis maps a signal $x(t)$ into a complex time-frequency transform $T_x(t,f)$

$$T_x(t,f) = \int x(u)\,\phi_{t,f}(u)^*\,\mathrm{d}u. \quad (5)$$

- $|T_x(t,f)|$ amplitude, $|T_x(t,f)|^2$ power, and $\arg[T_x(t,f)]$ phase of the time-frequency transform.
- S-transform (Stockwell et al., 1996) is a particular time-frequency transform: band-pass filter or a windowed Fourier transform with a Gaussian window whose width decreases with increasing frequency (standard deviation $1/|f|$).
- Since we deal with real signals, we apply the S-transform to the analytic signal $x_a(u)$ of $x(u)$.

Model of noisy data

- Each measured signal $x_n(t)$ is decomposed into a signal of interest $s_n(t)$ and a noise component $b_n(t)$

$$x_n(t) = s_n(t) + b_n(t) \qquad n = 1,\ldots,N \quad (6)$$

- The $b_n(t)$'s are N i.i.d. realizations of a noise $b(t)$ with zero mean and variance σ^2

Effect of noise on POWavg and avgPOW

$$\mathrm{E}\left[\mathrm{POWavg}_{x_{1:N}}(t,f)\right] = \mathrm{E}\left[\mathrm{POWavg}_{s_{1:N}}(t,f)\right] + \frac{1}{N}\mathrm{E}\left[|T_b(t,f)|^2\right] \quad (7)$$

$$\mathrm{E}\left[\mathrm{avgPOW}_{x_{1:N}}(t,f)\right] = \mathrm{E}\left[\mathrm{avgPOW}_{s_{1:N}}(t,f)\right] + \mathrm{E}\left[|T_b(t,f)|^2\right] \quad (8)$$

Case of color noise

- Power spectral density (PSD) approximately proportional to $1/\nu^c$ away from $\nu = 0$
- For the S-transform, we have approximately

$$\mathrm{E}\left[|T_b(t,f)|^2\right] \propto \frac{1}{f^{c-1}}. \quad (9)$$

- White noise ($c = 0$)

$$\mathrm{E}\left[|T_b(t,f)|^2\right] = \sigma^2\,\delta t \int |\phi_{t,f}(u)|^2\,\mathrm{d}u = \frac{|f|\sigma^2\,\delta t}{\sqrt{\pi}}. \quad (10)$$

3. Case of oscillatory signal

Model

- $s_n(t) = \Omega_n \cos(2\pi\nu_0 t + \phi_n)$ for $n = 1,\ldots,N$
- $\Omega_n \sim \mathcal{N}(\Omega_0, \tau_\Omega^2)$, $\phi_n \sim \mathrm{VonMises}(\phi_0, \kappa)$, with the Ω_n's and the ϕ_n's independent
- Noise $b(t)$ Gaussian with 0 mean and variance σ^2.

Results

$$\mathrm{E}\left[\mathrm{avgPOW}_{s_{1:N}}(t,f)\right] = \left(\Omega_0^2 + \tau_\Omega^2\right) e^{-(2\pi)^2\left(1-\frac{\nu_0}{f}\right)^2} \quad (3)$$

$$\mathrm{E}\left[\mathrm{POWavg}_{s_{1:N}}(t,f)\right] \approx \Omega_0^2 \rho^2 e^{-(2\pi)^2\left(1-\frac{\nu_0}{f}\right)^2} \quad (4)$$

Numerical example

- $\nu_0 \in \{40, 500\}$ Hz, $\Omega_0 = 1$, $\tau_\Omega = 0.1$, $\rho = 0.25$, $\delta t = 0.5$ ms, and $N = 300$ trials.
- Noise: $c \in \{0, 2\}$ (white or red), $\sigma^2 \in \{1, 10\}$

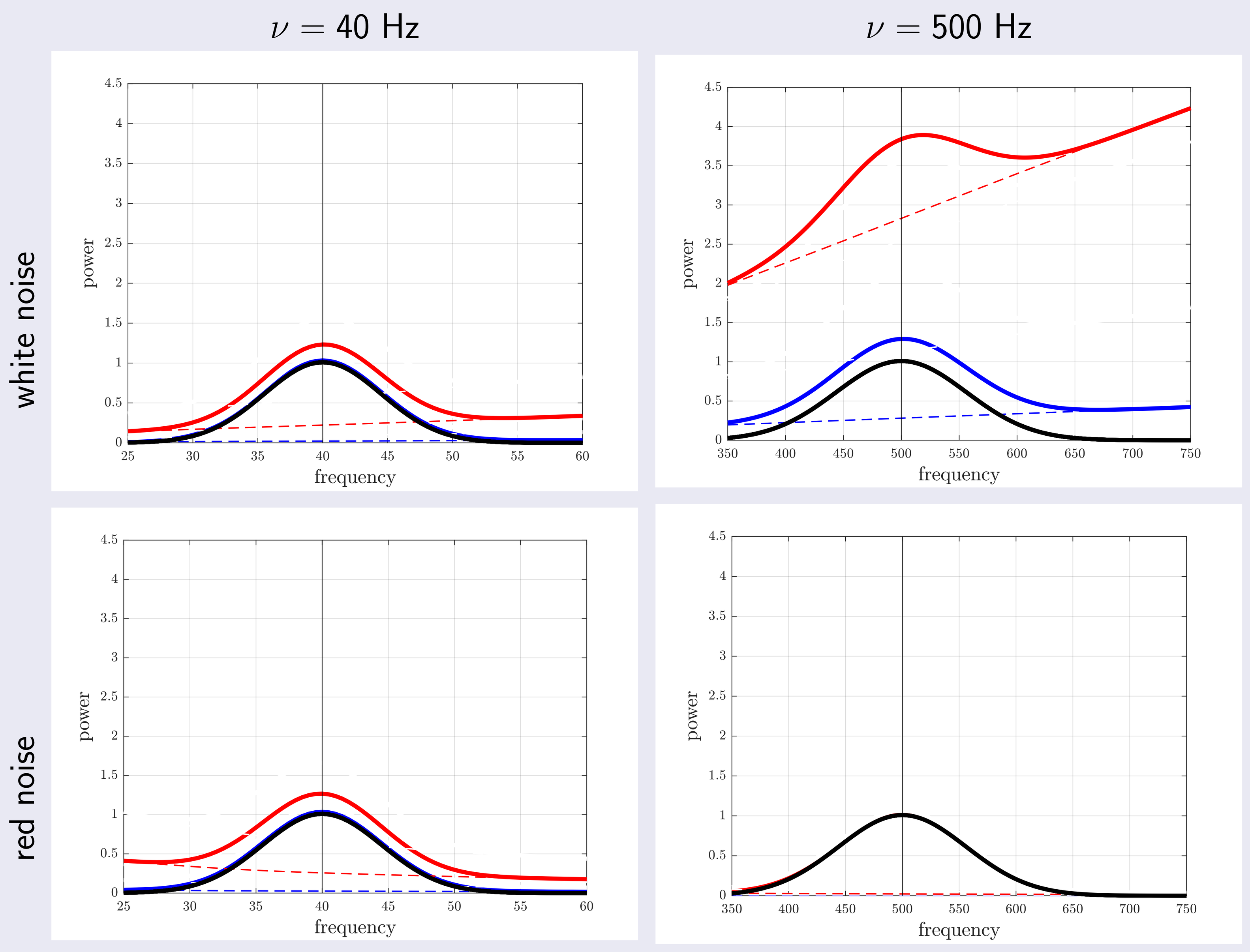

Figure : Effect of noise on $\mathrm{E}(\mathrm{avgPOW})$. $\mathrm{E}[|T_b(t,f)|^2]$ (dashed lines) and $\mathrm{E}(\mathrm{avgPOW})$, either for the original signal $s(t)$ (solid black line) or the noisy signal $x(t)$ (solid colored lines) for color noise, either white (top) or red (bottom), and variance equal to either $\sigma^2 = 1$ (blue) or $\sigma^2 = 10$ (red).

4. Simulation study

- Signals generated on a time window of $[-100, 100]$ ms, sampling rate $f_s = 2$ kHz ($\delta t = 0.5$ ms), different trials independent from one another
- For each trial n, oscillatory signal with amplitude Ω_n and frequency ν_0 as before
- Induced response in the $[20, 30]$ ms time window with phase $\phi_n^{(i)}$ von Mises distributed with concentration parameter $\kappa^{(i)} = 10$; ongoing activity the rest of the time with phase $\phi_n^{(o)}$ uniformly distribued
- White or red Gaussian noise with variance $\sigma^2 \in \{1, 10\}$
- Analysis with the S-transform
- Expected: influence of noise should be (i) larger on avgPOW than on POWavg; and (ii) larger for white noise than for red noise.

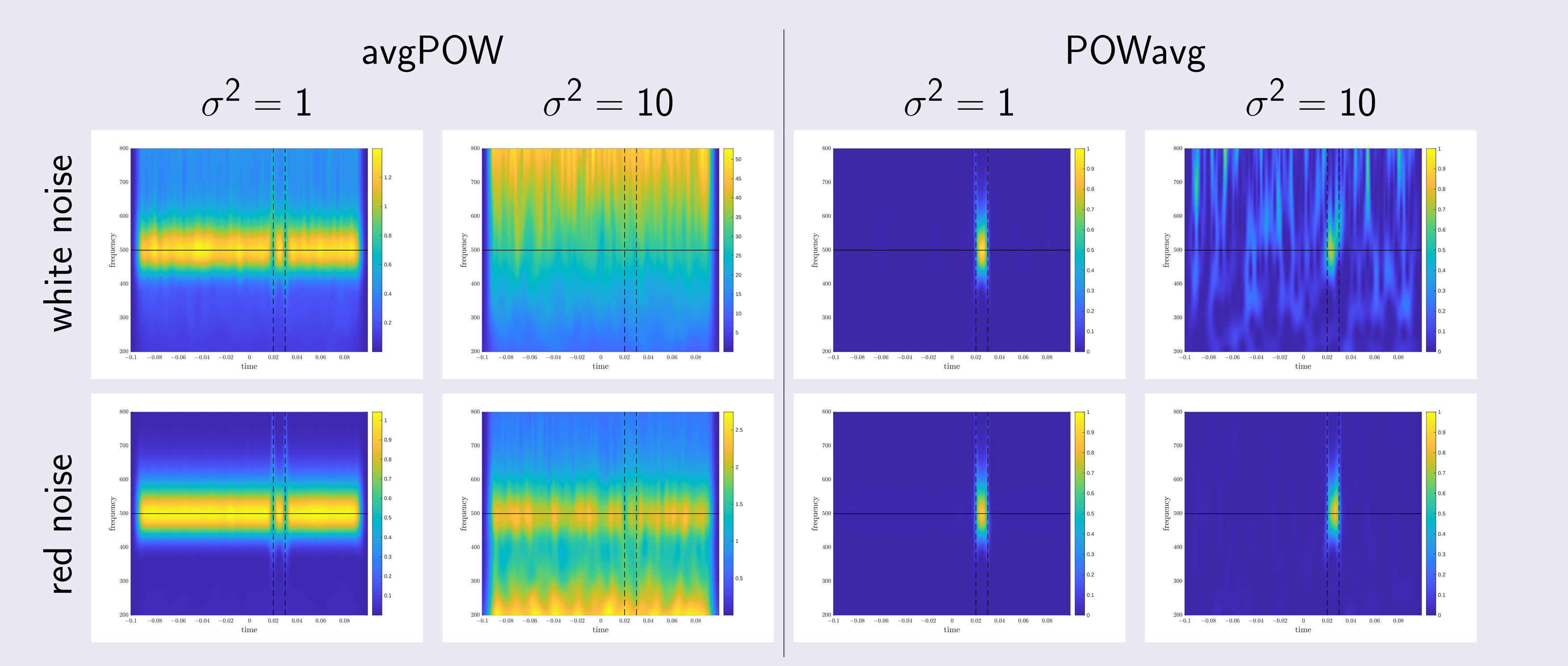

Figure : **Simulated data.** avgPOW and POWavg for a 500 Hz oscillatory signal with white noise (top) or red noise (bottom) either with $\sigma^2 = 1$ or $\sigma^2 = 10$. For avgPOW, the color scales differ for all plots, while they are identical for POWavg.

5. Conclusion

- Additive noise tends to increase both avgPOW and POWavg on average
- The main factors of influence for both measures are the noise variance, the number of trials, the sampling rate, the type of noise, and the frequency of interest
- An increasing number of trials reduces the influence of noise on POWavg but not on avgPOW
- The type of time-frequency transform (e.g, S-transform or continuous wavelet transform) has an influence on the way the time-frequency transforms of both the signal and the noise behave as a function of frequency
- In the case of a color noise with PSD of the form $1/\nu^c$ analyzed with the S-transform, the relative effect of noise (i) increases with increasing frequency for $c < 1$; (ii) does not depend on frequency for $c = 1$; and (iii) decreases with increasing frequency for $c > 1$
- Effects were established using theoretical calculations and simulation study
- What about inter-trial coherence (ITC)? (Benhamou et al., 2023)

References

Benhamou, J., Le Van Quyen, M., Marrelec, G., 2023. Time-frequency analysis of event-related brain recordings: Connecting power of evoked potential and inter-trial coherence. IEEE Trans. Biomed. Engineering 70, 1599–1610.

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