Supplementary material #2 for manuscript "Time-frequency analysis of event-related brain recordings: Connecting power of evoked potential and inter-trial coherence"

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1 Supplementary figure for simulation results presented in manuscript

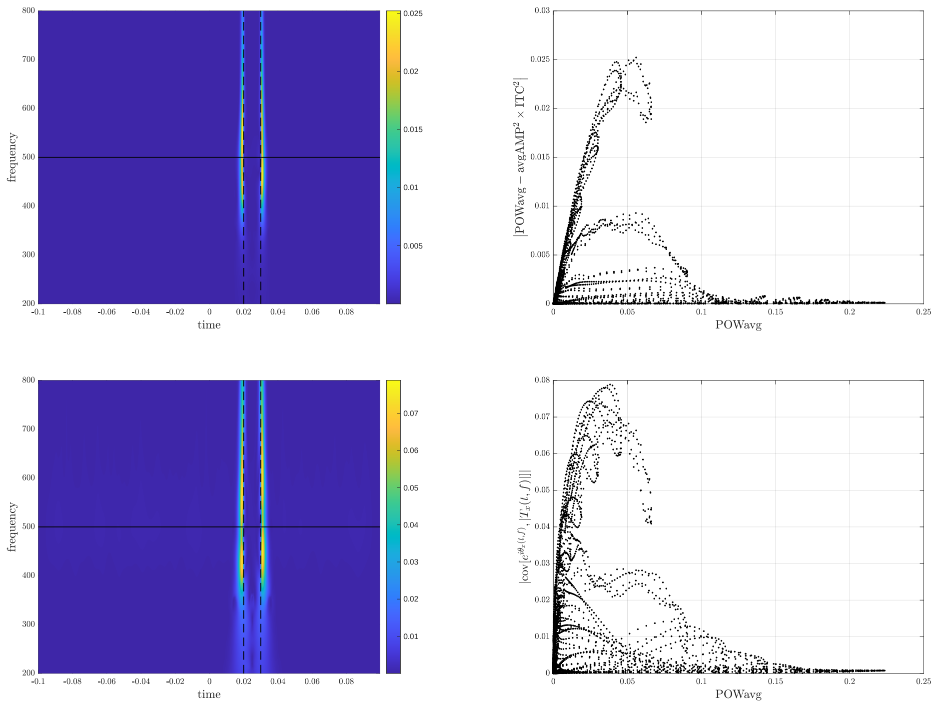

Figure 1: **Simulation study.** Results from time-frequency transform with the S-transform for signals with $\kappa^{(i)} = 10$ and $\tau_\nu = 0$. Quantification of $|\mathrm{POWavg} - \mathrm{avgAMP}^2 \times \mathrm{ITC}^2|$ (top) and $|\mathrm{Cov}[e^{i\theta_x(t,f)}, |T_x(t,f)|]|$ (bottom).

2 Supplementary figures for experimental results presented in manuscript

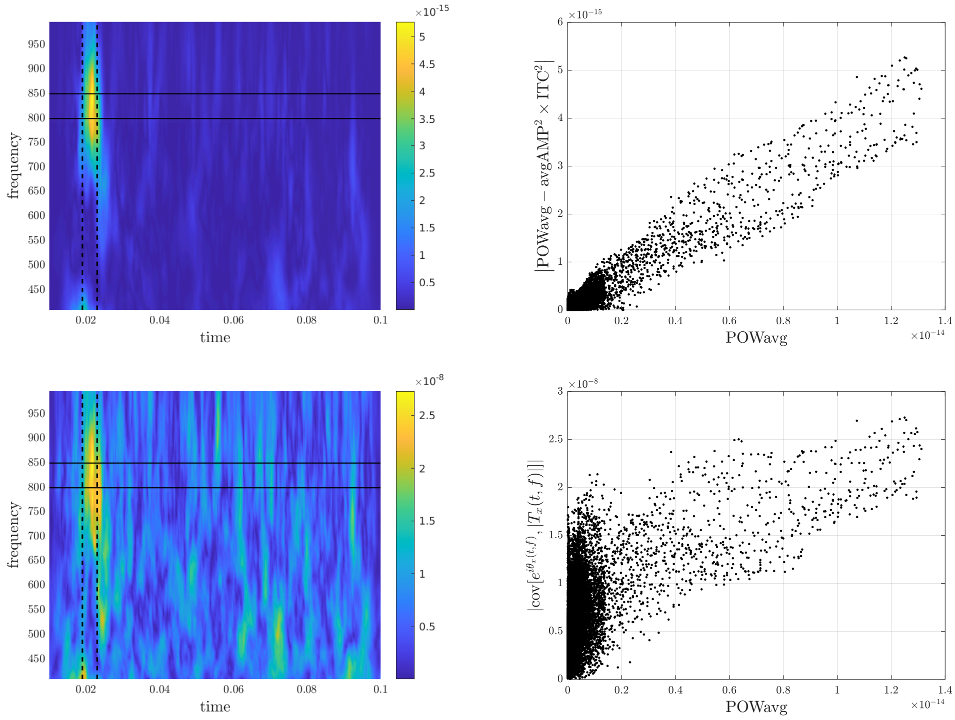

Figure 2: **Real data.** Results from time-frequency transform with the S-transform. Quantification of $|\text{POWavg} - \text{avgAMP}^2 \times \text{ITC}^2|$ (top) and $|\text{Cov}[e^{i\theta_x(t,f)}, |T_x(t,f)|]|$ (bottom).

3 Results of all simulations with $\nu_0 = 500$ Hz

3.1 S-transform

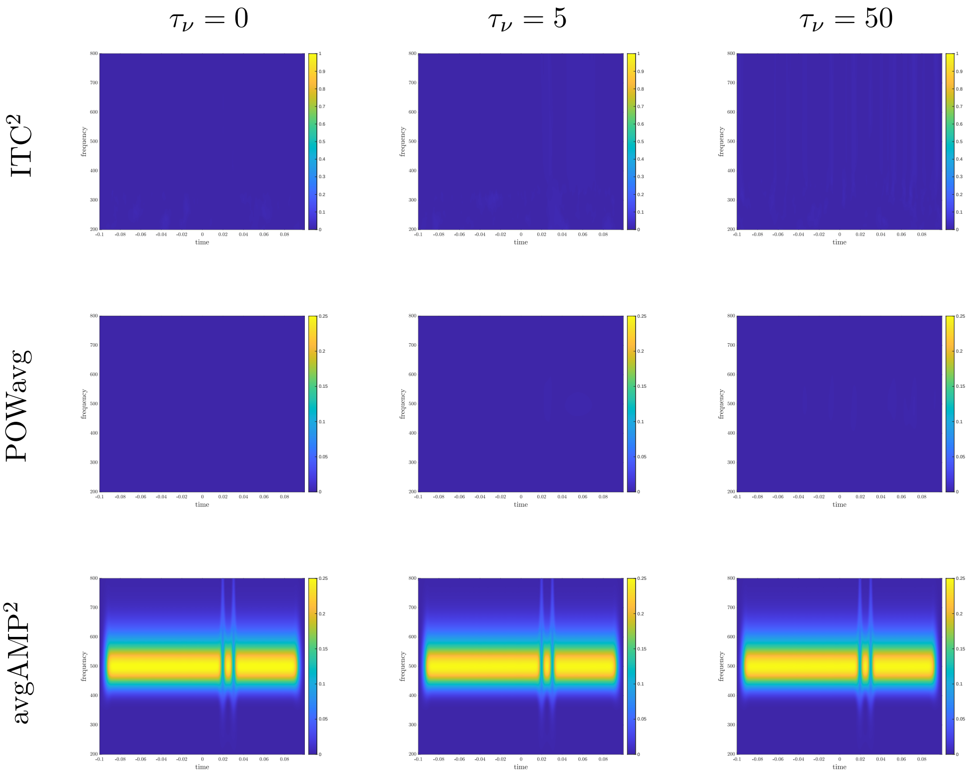

Figure 3: **Simulation study.** Results for the S-transform. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 0$ (which corresponds to a squared mean resultant length of 0) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2/4 = 0.25$.

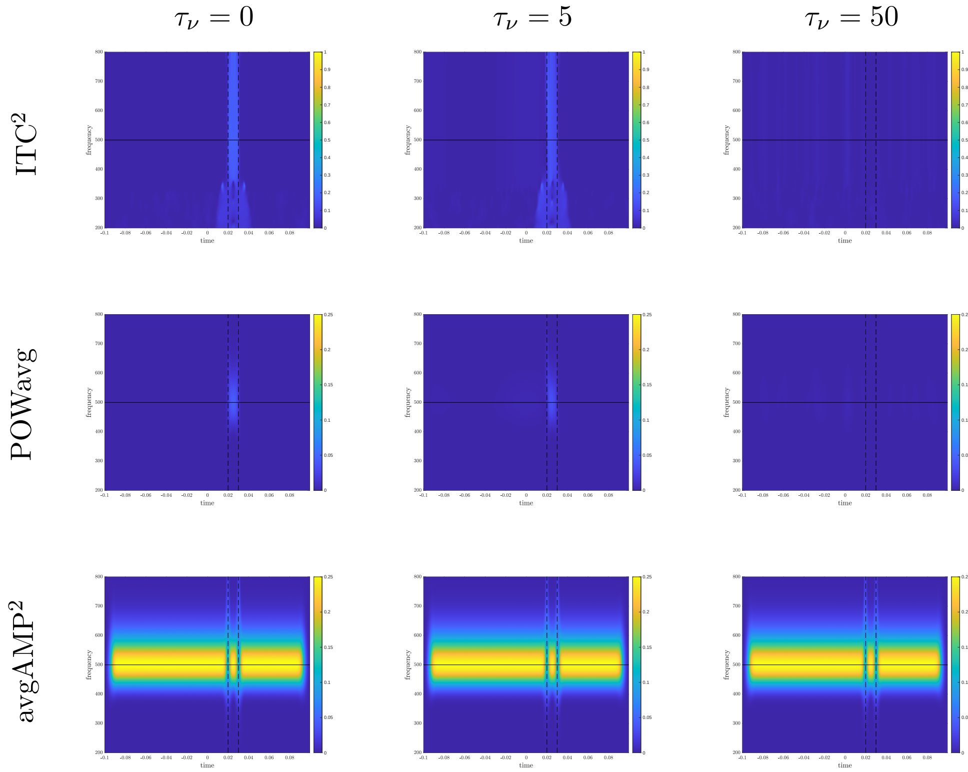

Figure 4: **Simulation study.** Results for the S-transform. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 1$ (which corresponds to a squared mean resultant length of 0.199) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2/4 = 0.25$.

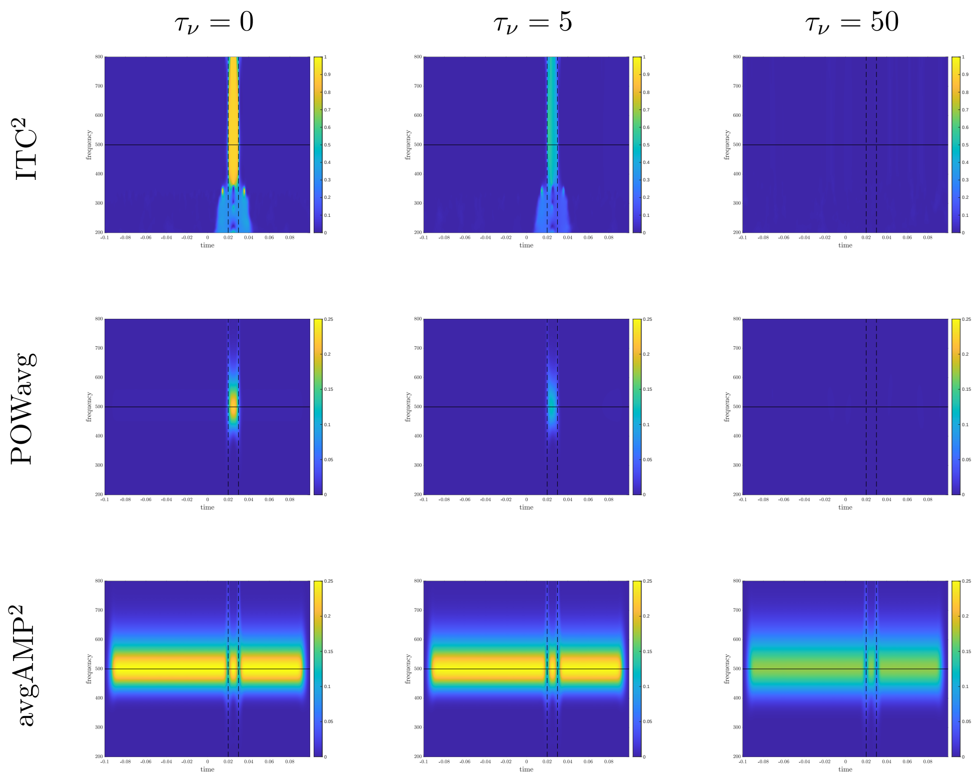

Figure 5: **Simulation study.** Results for the S-transform. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 10$ (which corresponds to a squared mean resultant length of 0.900) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2/4 = 0.25$.

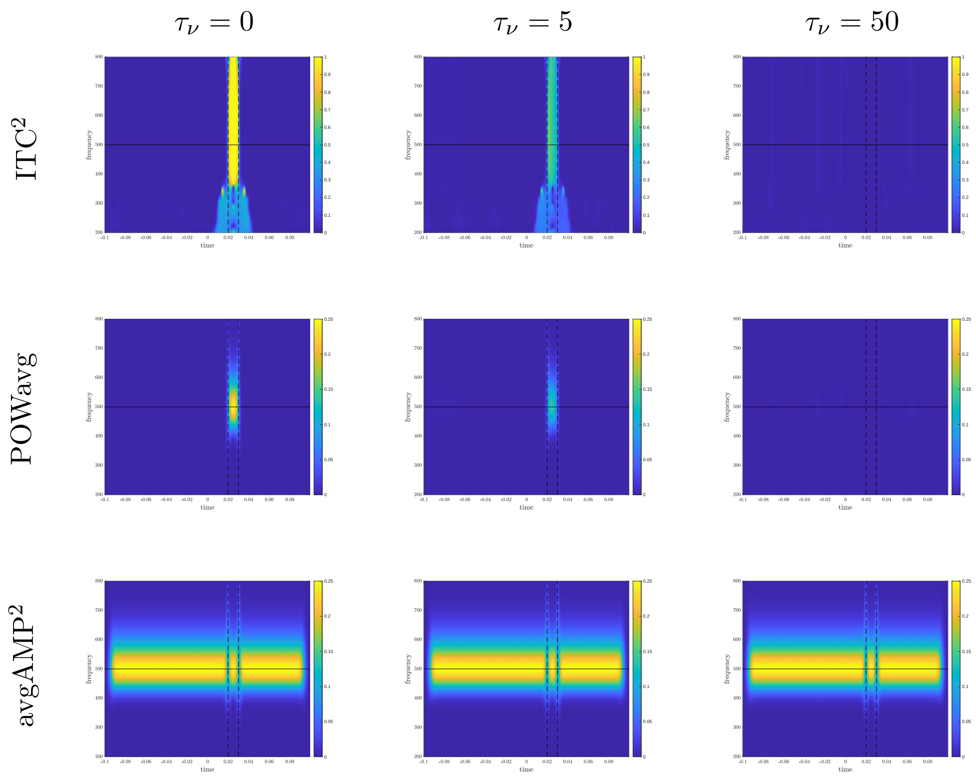

Figure 6: **Simulation study.** Results for the S-transform. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 100$ (which corresponds to a squared mean resultant length of 0.990) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2/4 = 0.25$.

3.2 Morlet wavelet

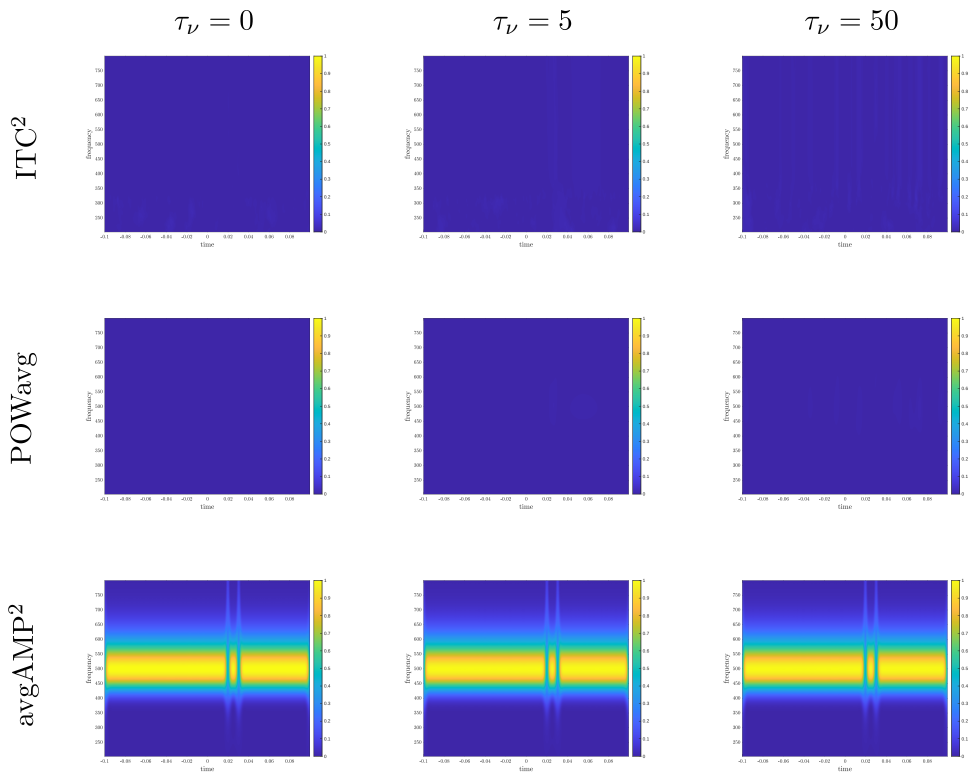

Figure 7: **Simulation study.** Results for the wavelet transform with analytic Morlet wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 0$ (which corresponds to a squared mean resultant length of 0) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

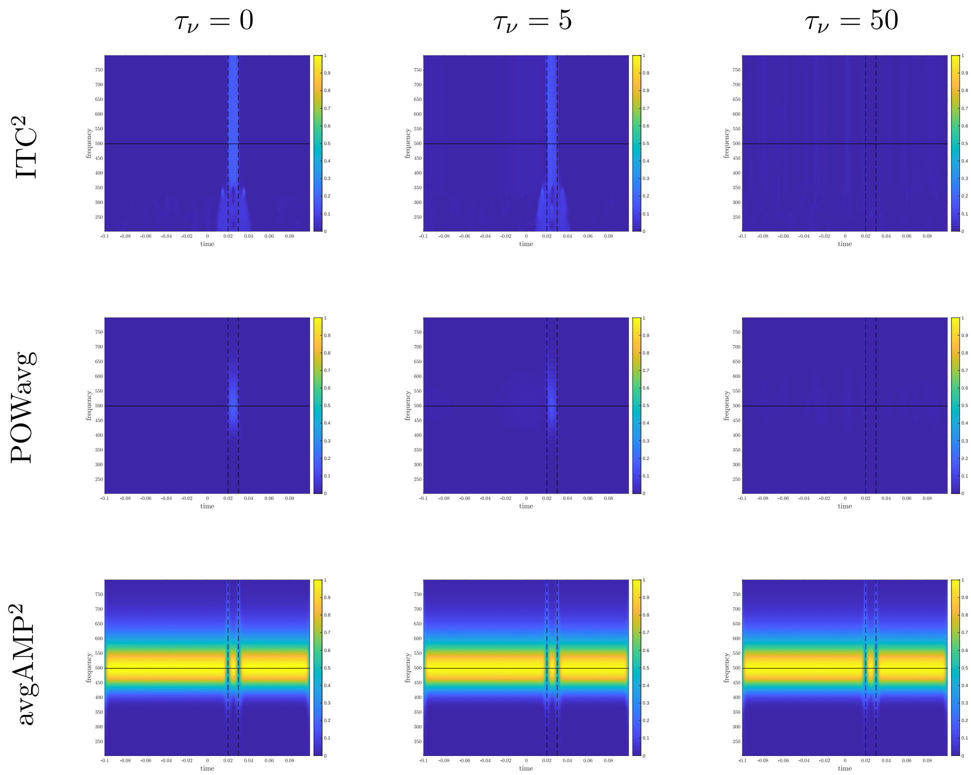

Figure 8: **Simulation study.** Results for the wavelet transform with analytic Morlet wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 1$ (which corresponds to a squared mean resultant length of 0.199) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

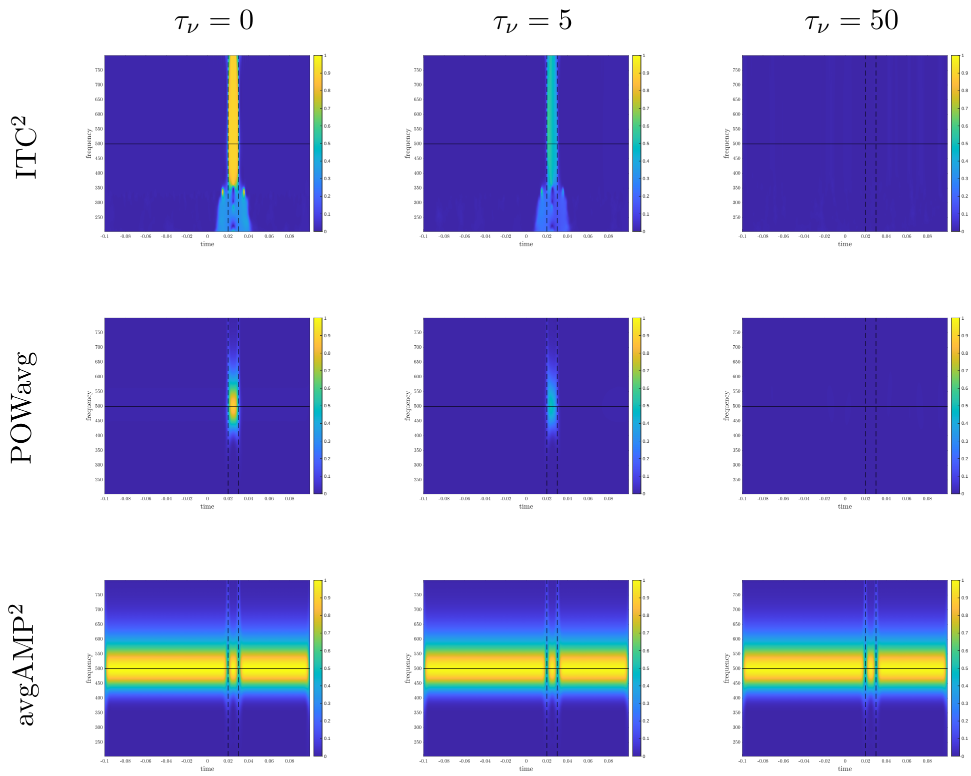

Figure 9: **Simulation study.** Results for the wavelet transform with analytic Morlet wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 10$ (which corresponds to a squared mean resultant length of 0.900) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

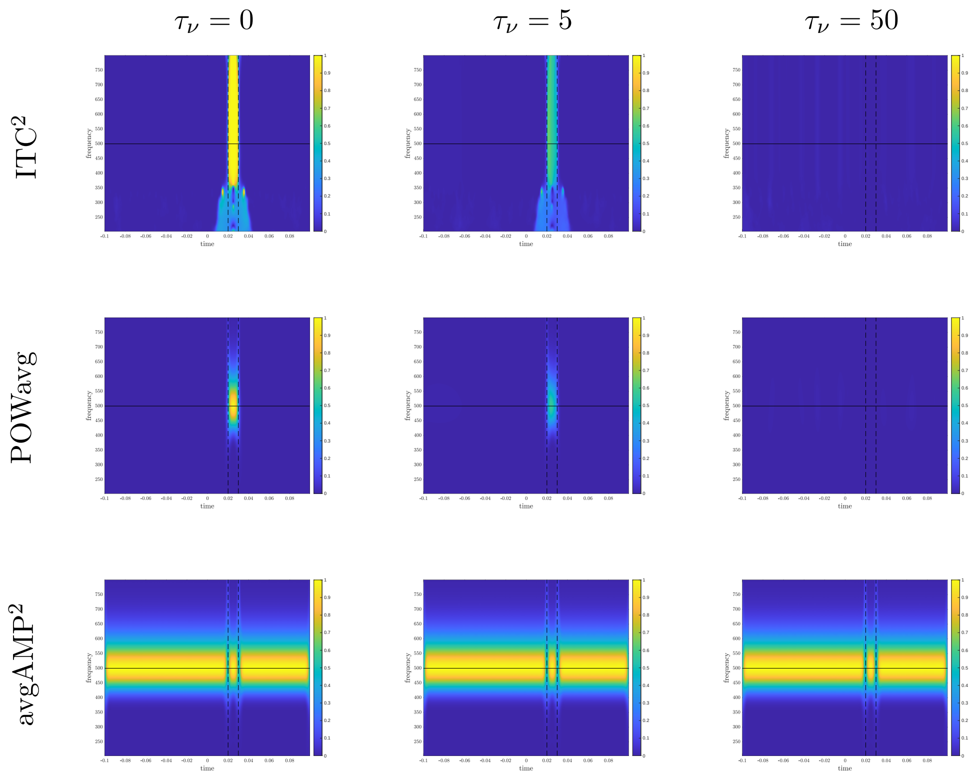

Figure 10: **Simulation study.** Results for the wavelet transform with analytic Morlet wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 100$ (which corresponds to a squared mean resultant length of 0.990) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

3.3 Morse wavelet

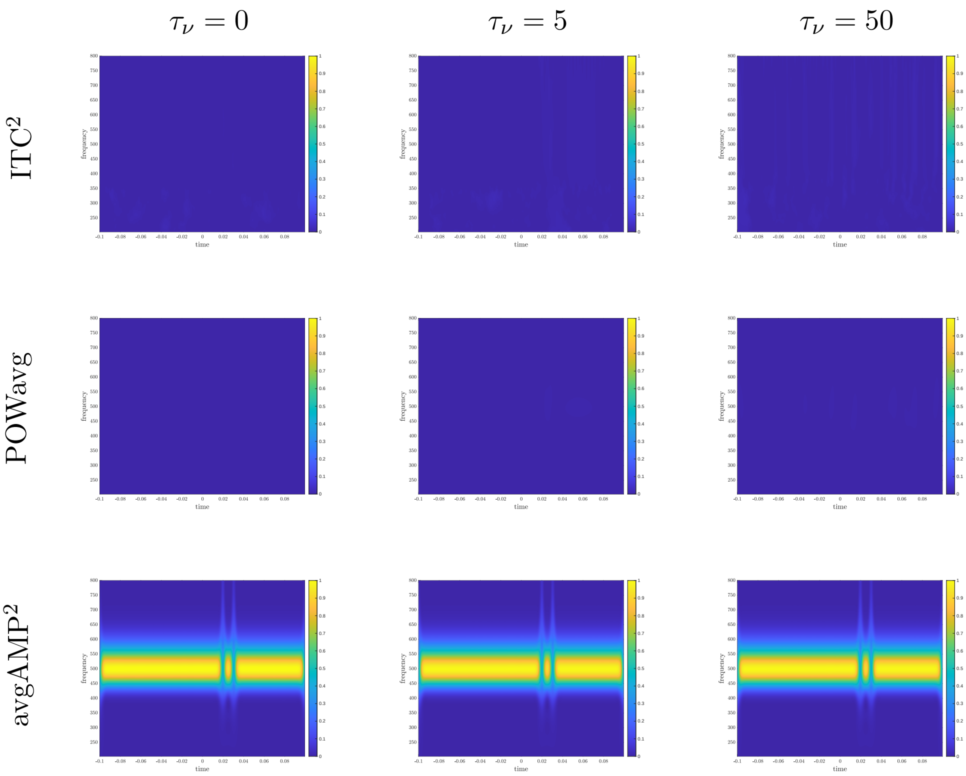

Figure 11: **Simulation study.** Results for the wavelet transform with Morse wavelet. ITC^2 (top), POWavg (middle) and $avgAMP^2$ (bottom) for signals with concentration parameter $\kappa = 0$ (which corresponds to a squared mean resultant length of 0) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

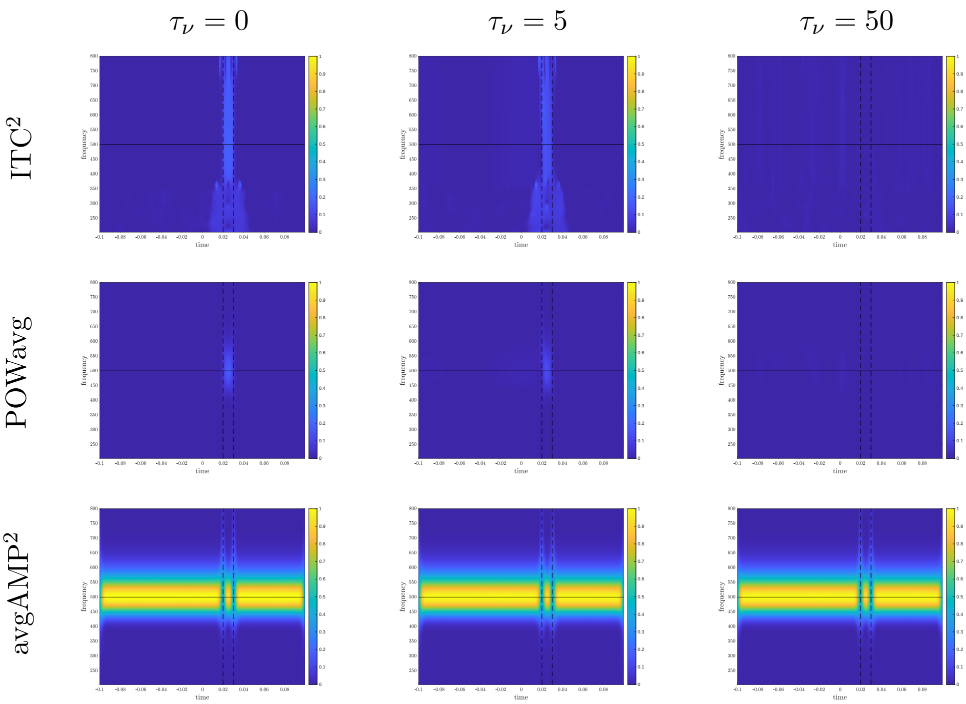

Figure 12: **Simulation study.** Results for the wavelet transform with Morse wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 1$ (which corresponds to a squared mean resultant length of 0.199) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

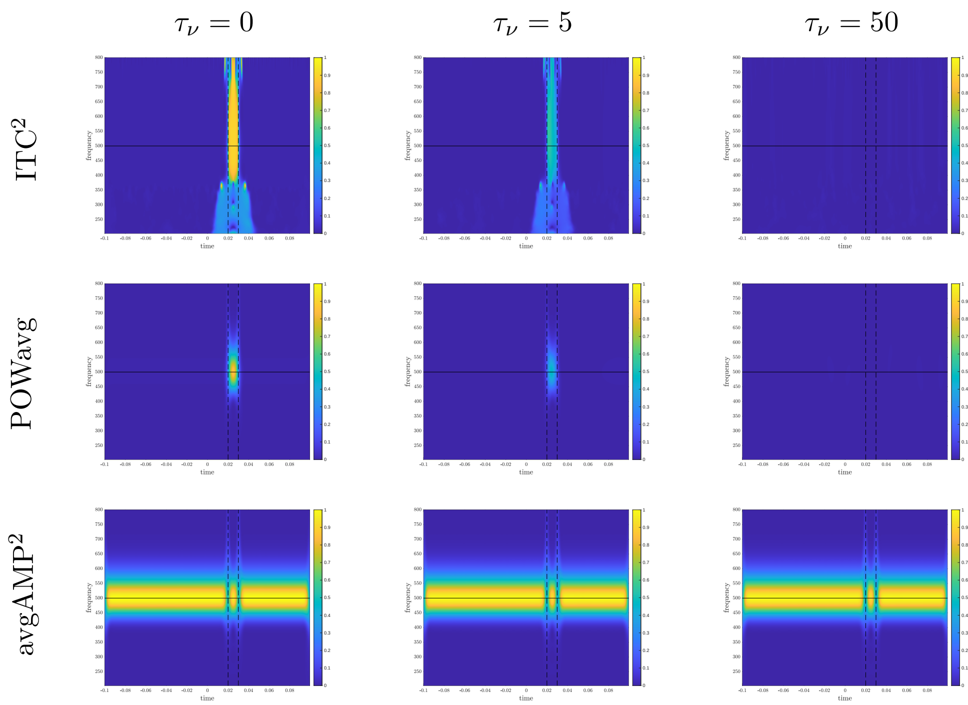

Figure 13: **Simulation study.** Results for the wavelet transform with Morse wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 10$ (which corresponds to a squared mean resultant length of 0.900) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

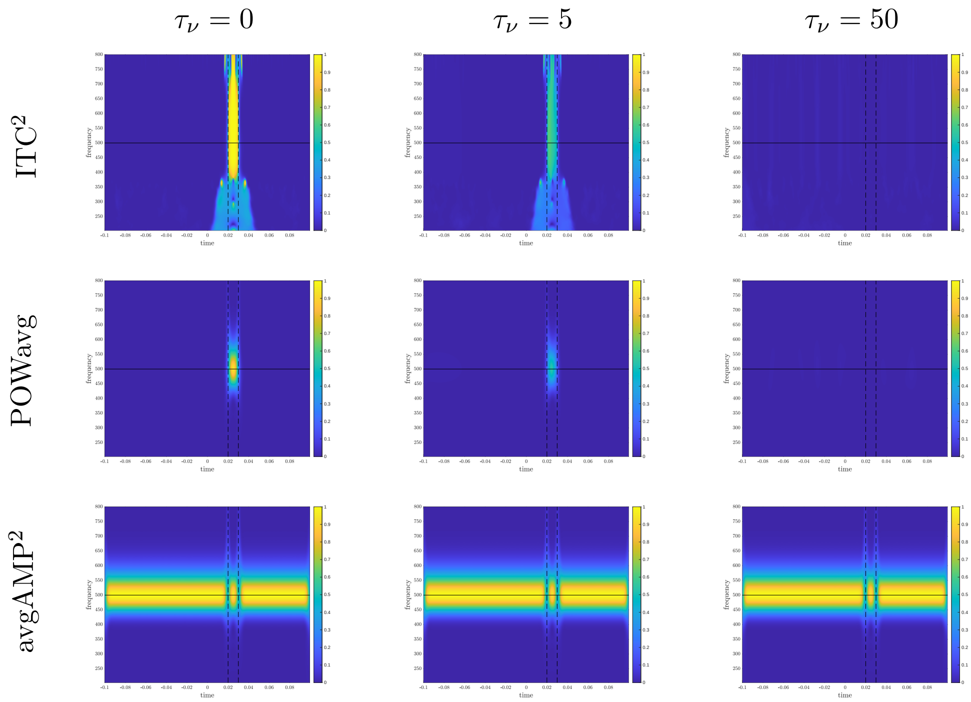

Figure 14: **Simulation study.** Results for the wavelet transform with Morse wavelet. ITC^2 (top), POWavg (middle) and avgAMP^2 (bottom) for signals with concentration parameter $\kappa = 100$ (which corresponds to a squared mean resultant length of 0.990) and $\tau_\nu \in \{0, 5, 50\}$. At the signal frequency, the expected value of the energy of the time-frequency transform is $\Omega_0^2 = 1$.

4 Simulation with $\nu_0 = 40$ Hz

4.1 Data generation and analysis

We applied a framework similar to the one described in the simulation section (Section III) of the manuscript. We used the following parameter values: signal generated on a time window of $[-1\,250, 1\,250]$ ms; induced response in the $[250, 375]$ ms time window; $f_s = 200$ Hz (corresponding to $\delta t = 5$ ms); $N = 300$ trials; same distribution for the Ω_n's as in Table II of the manuscript; $\nu_0 = 40$ Hz and $\tau_\nu = 0$; same distribution for the $\phi_n^{(o)}$'s as in Table II of the manuscript; same distribution for the $\phi_n^{(i)}$'s as in Table II of the manuscript with $\kappa = 10$ only; same noise standard deviation. We applied the S-transform.

4.2 Results

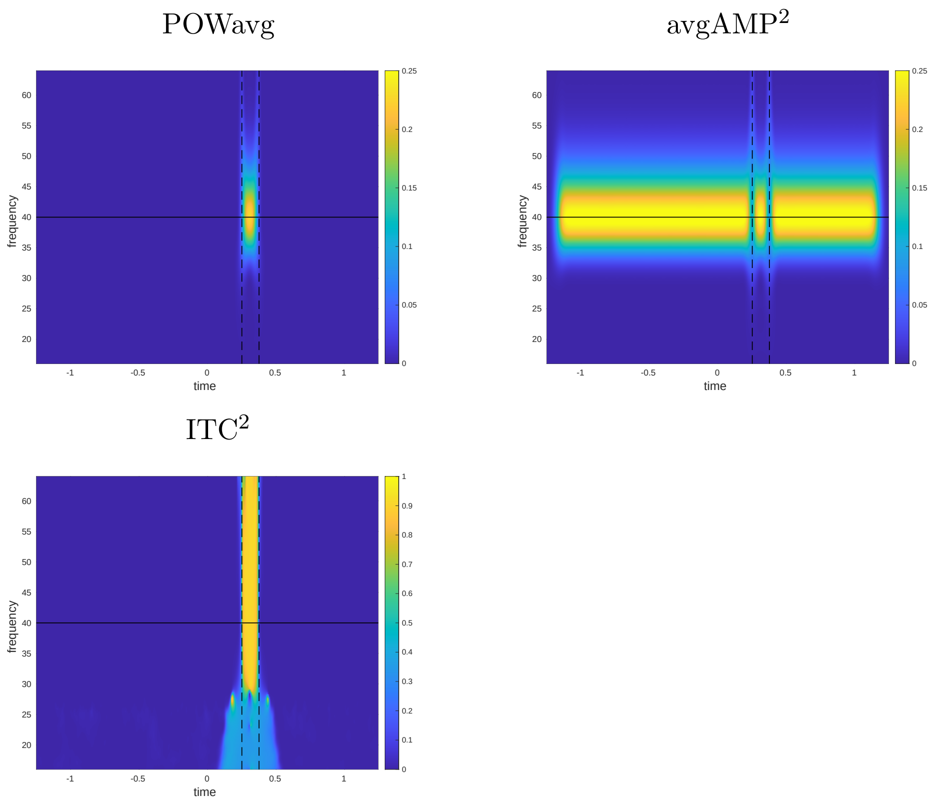

Figure 15: **Simulation study.** POWavg, avgAMP^2, and ITC^2 from the S-transform of a 40 Hz signal.