BonHom documentation (version 1.3)

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1 Modifications

- Jan, 8th, 2015. Creation of BonHomV1p3. One minor bug has been corrected in BonHomV1p2: lines 64-65 changes have been made regarding the definition of R and ϕ. The bug which has been corrected introduced a very small error in the calculated stiffness coefficients. The documentation below and numerical examples have not been updated.

2 Introduction

`BonHom` is a Matlab function which evaluates the homogenized linear elastic properties of a composite. The function is limited to the evaluation of the transversely isotropic (TI) effective properties of a composite material consisting of a cylindrical TI fibers of circular cross section embedded periodically in a TI matrix (Fig. 1). The plane of isotropy of the TI materials of the fiber and matrix is perpendicular to the fibers axis. Unit cells are hexagonal.

The program is based on the derivation of effective elastic properties via the method of asymptotic homogenization [2]. The classical approach is used where the effective properties are defined in terms of a solution to the cell problem. Various methods may be employed to solve this cell problem. Here we use multipole expansions in doubly-periodic functions. Because of this we must truncate such expansions at some specified order, we call this the order of the multipole expansion. In general this number has to be higher when the volume fraction of the inclusion phase (fiber) is larger. In most

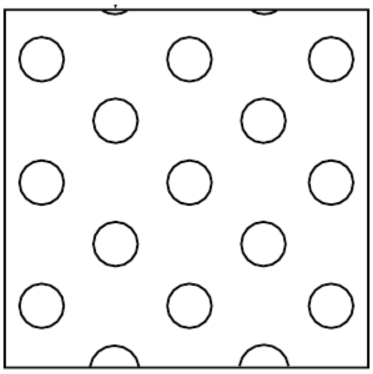

Figure 1: Represention of the cross-section of the composite with a periodic distribution of fibers.

practical cases this number is rather small, we use a default of 5 in `BonHom` code. The homogenization method is exact for periodic media provided that the order of the multipole expansion is large enough. The method has been initially designed to model the effective properties of compact bone at the millimeter scale [1].

3 Conditions of use

This program is citation-ware. If you are publishing any work, where this program or any part of it has been used, you must reference the paper shown below and mention the name of the program in the publication. The authors of the program do not offer any support for this product whatsoever. The program is offered free of charge. It may be distributed and freely modified. It is, however, not permitted to utilize any part of the software in commercial products without prior written consent of the authors.

MATLAB is a trademark of The MathWorks, Inc. Trademarks of the companies men- tioned in this documentation appear for identification purposes only and are the property of their respective companies.

Paper to reference :

Parnell, W.J. and Grimal, Q. 'The influence of mesoscopic porosity on cortical bone anisotropy. Investigations via asymptotic homogenization' Journal of the Royal Society Interface 6 (2009) 97-109

4 Run BonHom

The input data are the elastic properties of the matrix and fiber, and the volume fraction of the fiber.

- `input.Cm` = tensor in matrix form for the matrix
- `input.Cf` = tensor in matrix form for the fiber
- `input.vf` = vector of volume fractions of fiber ranging from 0 to 1
- `option.nterms` = number of terms in the multipole expansion

The notation convention for TI properties is the following:

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{pmatrix}$$

where $C_{11} = C_{22}$, $C_{13} = C_{23}$ and $C_{66} = \frac{C_{11}-C_{12}}{2}$ are the shear coefficients. The defaults number of terms in the multipole expansion is 5. The user should check for each problem which number is appropriate. The solution should converge with sufficient number of terms. Implemented number of terms are 3, 5, 7, 9 and 11. If the number of terms is too high, the linear system to solve for the cell problem may by ill-conditionned. This however does not affect the accurracy of the solution in our experience.

The command to run the program is:

```
[output]= BonHomV1p2 (input,option).
```

The output is a Matlab structure which contains the stiffness tensor for each fiber volume fraction.

The Matlab datafile `ForceStruc.mat` contains data necessary for the computation and should be in Matlab path.

See the example file for more details.

5 Validation and limitations

The solutions calculated by the program have been validated against other analytical solutions and bounds. A comparison with the Mori-Tanaka solution is provided in Figure 2 at relatively low fiber volume fraction. The program certainly yields reliable results at least up to a volume fraction of 50%. The program may also be appropriate to solve for higher volume fractions but in those cases, a carefull analysis of the optimim number of expansion terms should be done.

`BonHom` will not give results for a perfect fluid in the pores (i.e. $\mu = 0$), however taking an arbitrary small value of $\mu = 0$ should yield a correct result.

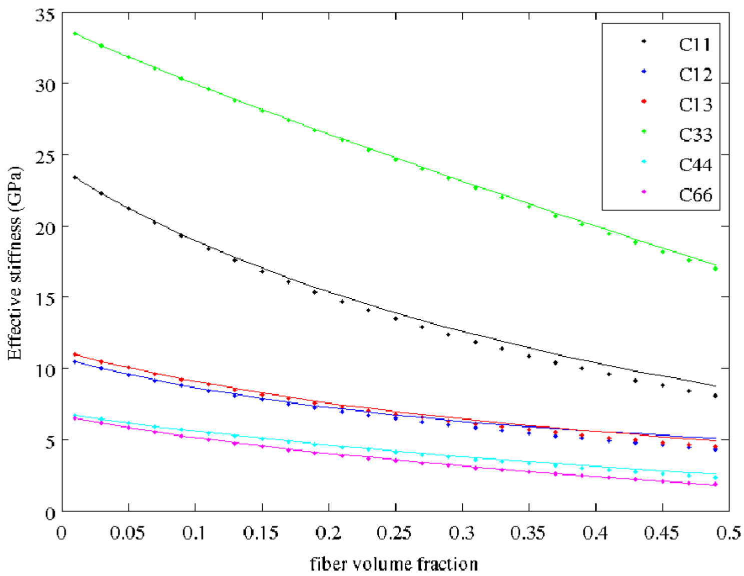

Figure 2: Mori-tanaka (dots) and AS solution (continuous line) for the material parameters as given in the example file.

Matlab will give warnings such as 'Matrix is close to singular...' for values of porosity very close to zero, but the results should nevertheless be valid.

6 References

[1] W. J. Parnell and Q. Grimal. The influence of mesoscale porosity on cortical bone anisotropy. Investigations via asymptotic homogenization. J R Soc Interface, 6(30):97-109, 2009.

[2] W. J. Parnell and I. D. Abrahams Homogenization for wave propagation in periodic fibre-reinforced media with complex microstructure. I - Theory. JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS, 56(7):2521-2540, 2008.